De Giuseppe Theorem (Extended): Principle of Microscopic and Macroscopic Entanglement

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Abstract

We extend the De Giuseppe short formulation and present a mathematically precise criterion for when informational and configurational constraints (the *matrioskas* (ΔC), (ΔM), (ΔL)) produce entanglement-like correlations at macroscopic scales. After formal definitions we provide quantitative diagnostics (mutual information and logarithmic negativity for Gaussian realizations) and a constructive toy model (coupled oscillator networks with mediated preparation) that demonstrates how a preparation protocol can produce a non-separable global state. We state an operational theorem linking admissible configuration sets to measurable non-factorizability and outline experimental parameters and robustness conditions. Nima's proto-structural viewpoint [1] is used as foundational motivation.

1 Introduction

The proto-structural matrioska idea arranges constraints in nested layers denoted symbolically as

$$(\Delta C) \leftrightarrow (\Delta M) \leftrightarrow (\Delta L),$$

where (ΔC) encodes geometric admissibility, (ΔM) material microstate coherence, and (ΔL) informational correlations [1]. We formalize how a restriction of global configuration space by these layers can imply non-factorizability of local descriptions, i.e. entanglement-like correlations, and provide explicit diagnostics and a constructible model to demonstrate the principle.

2 Formal framework

Let Γ be the space of global configurations (histories / state assignments) for a composite system partitioned into subsystems A, B. Define projection maps

$$\Phi_C : \Gamma \to \mathcal{C}, \quad \Phi_M : \Gamma \to \mathcal{M}, \quad \Phi_L : \Gamma \to \mathcal{L},$$

which extract the (ΔC), (ΔM), (ΔL) data respectively. Introduce an admissible set

$$\mathcal{A}(\mathcal{C},\mathcal{M},\mathcal{L}) = \{\gamma \in \Gamma \mid \Phi_C(\gamma) \in \mathcal{C},\ \Phi_M(\gamma) \in \mathcal{M},\ \Phi_L(\gamma) \in \mathcal{L}\},$$

and the configuration indicator

$$f(\gamma) = \mathbf{1}_{\mathcal{A}}(\gamma) = \begin{cases} 1 & \gamma \in \mathcal{A}, \\ 0 & \gamma \notin \mathcal{A}. \end{cases}$$

Definition 2.1 (Non-factorizability)**.** Let μ be a probability (or density) on Γ supported on $\mathcal{A}$. Denote marginals μ_A, μ_B. We say the joint law is ε-nonfactorizable if

$$D(\mu, \mu_A \otimes \mu_B) := \inf_{\nu_A,\nu_B} \|\mu - \nu_A \otimes \nu_B\|_1 > \varepsilon,$$

where $\|\cdot\|_1$ is total variation (or trace distance in quantum formalism).

Operationally, non-factorizability implies measurable correlations: mutual information

$$I(A:B) = S(\mu_A) + S(\mu_B) - S(\mu_{AB}) > 0,$$

and in quantum Gaussian realizations one can test negativity via logarithmic negativity E_N (see below).

3 De Giuseppe theorem (operational form)

Theorem 3.1 (Operational De Giuseppe Theorem)**.** *Suppose there exists a preparation protocol P that produces, with nonzero probability, a configuration $\gamma \in \mathcal{A}(\mathcal{C},\mathcal{M},\mathcal{L})$. If the induced joint state μ on Γ satisfies*

$$D(\mu, \mu_A \otimes \mu_B) > \varepsilon > 0,$$

then A and B exhibit operationally detectable entanglement-like correlations. Furthermore, for Gaussian continuous-variable realizations, this condition can be certified by the partial-transpose criterion: if the smallest symplectic eigenvalue $\tilde{\nu}_- < \frac{1}{2}$ (in units $\hbar = 1$), then the state is non-separable and logarithmic negativity

$$E_N = \max(0, -\ln(2\tilde{\nu}_-)) > 0.$$

Remark 3.1. This theorem is *conditional*: it reduces the problem of macroscopic entanglement to (i) existence of a preparation P producing $\gamma \in \mathcal{A}$ and (ii) demonstration that the induced state is non-factorizable beyond noise thresholds. The theorem is agnostic on the microscopic mechanism (topological identification, mediated coupling, or explicit encoding); it only requires that the matrioska constraints define a nontrivial admissible region $\mathcal{A}$.

4 Toy constructive model: coupled oscillator networks

We present an explicit toy model where the theorem is constructive. Consider two macroscopic subsystems A, B each modeled as a collection of N harmonic modes (index i), with canonical variables $q_i^{(A)}, p_i^{(A)}$ and $q_i^{(B)}, p_i^{(B)}$. Introduce a mediating global mode m (optical or collective mechanical mode) used only during preparation.

Hamiltonian (units $\hbar = 1$):

$$H = H_A + H_B + H_m + H_{\text{int}}(t),$$

with

$$H_A = \sum_{i=1}^{N} \frac{\omega_i}{2}\big((p_i^{(A)})^2 + (q_i^{(A)})^2\big), \quad H_m = \frac{\Omega}{2}(P_m^2 + Q_m^2),$$

and time-dependent interaction used in preparation interval $t \in [0, t_p]$:

$$H_{\text{int}}(t) = g_A(t) Q_m \sum_i \kappa_i q_i^{(A)} + g_B(t) Q_m \sum_j \kappa_j q_j^{(B)}.$$

Choose g_A, g_B pulses that entangle m with a collective mode of A, then swap correlation to B and finally decouple m. Standard Gaussian-state evolution (linear coupling) yields after t_p a two-mode (collective) covariance matrix σ_{AB}. One computes symplectic spectrum and checks $\tilde{\nu}_- < \frac{1}{2}$ indicating entanglement between collective degrees of freedom of macrosystems A, B.

Key point: after preparation, the mediator m can be turned off and removed: the resulting entanglement is a property of the prepared state (a consequence of (ΔL) encoding and the controlled (ΔM) coherence). The energetic cost is concentrated in preparation; maintenance requires appropriate isolation (reducing decoherence rates).

5 Diagnostics and robustness

For Gaussian states the logarithmic negativity E_N provides a quantitative witness, while mutual information $I(A : B)$ gives an operational (classical) measure robust to coarse-graining. Robustness requires decoherence time t_{dec} large compared to readout time t_{read}. Let noise channel $\mathcal{E}_\eta$ act with strength η; one must ensure $E_N(\mathcal{E}_\eta(\sigma_{AB})) > 0$. Practically, this imposes bounds on tolerances $\delta_C, \delta_M, \delta_L$ in the matrioska layers.

6 Experimental platforms and estimates

Promising platforms for a constructive proof-of-principle include:

- Optomechanical arrays: collective mechanical modes coupled to an optical bus.
- Superconducting circuits: microwave resonators acting as mediators among lumped-element macroscopic modes.
- Engineered condensed-matter devices: topological modes providing global constraints.

Each platform offers established techniques to prepare Gaussian entangled states across macroscopic degrees of freedom with quantifiable decoherence — enabling direct tests of the theorem.

7 Conclusions and roadmap

We provided a compact but quantitative extension of the De Giuseppe idea: (i) a precise operational theorem linking matrioska-admissible configurations to measurable non-factorizability, (ii) explicit diagnostics (mutual information, logarithmic negativity), and (iii) a constructive toy model showing how a preparation protocol can realize macroscopic entanglement. To move from principle to incontrovertible demonstration: (A) pick platform, (B) model $\Gamma, \mathcal{A}$ explicitly, (C) design P and compute entanglement measures, (D) implement and measure. Successful execution would convert the present conditional theorem into an experimentally verified principle.

References

[1] Nima, M. (2026). *(ΔC) $\leftrightarrow$ (ΔM) $\leftrightarrow$ (ΔL): Cosmology from Chaos Substrate via Matryoshka Filtering and Kakeya Stability.* Zenodo. `https://doi.org/10.5281/zenodo.18148819`